Performance Analysis of OLSR Multipoint Relay Flooding in Two Ad Hoc Wireless Network Models

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Abstract: We analyze the performance of *ad hoc* pro-active routing protocol OLSR. In particular we focuse on the multipoint relay concept which is the most salient feature of this protocol and which brings the most significant breakthrough in performance. We will anlyse the performances in two radio network models: the random graph model and the unit graph. The random graph is more suitable for indoor networks, and the unit graph is more suitable for outdoor networks. We compare the performance of OLSR with the performance of link state protocols using full flooding, such as OSPF.

Key-words: Wireless network, mobile ad-hoc networks, flooding, mulitpoint relays, random graphes.

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Analyses des performances de l'inondation par relais multipoints dans deux modèles de réseaux aléatoires

Résumé : Nous analysons les performances du protocole de routage *ad hoc* OLSR. En particulier nous nous intéressons au concept des relais multipoints qui constituent l'innovation la plus importante de ce protocole et lui apportent le principal gain en performance. Nous évaluons les performances dans deux modèles de réseaux radio: le graphe aléatoire et le graphe unité. Le graphe aléatoire convient mieux aux réseaux d'intérieur. Le graphe unité s'adresse davantage aux réseaux d'extérieur. Nous comparons les performances de OLSR avec les performances des protocoles d'état des liens à inondation totale, comme OSPF.

Mots-clés : Réseau sans fil, réseaux mobile ad-hoc, inondation, multipoint relai, graphes aléatoires.

1 Introduction

Radio networking is emerging as one of the most promising challenge made possible by new technology trends. Mobile Wireless networking brings a new dimension of freedom in internet connectivity. Among the numerous architectures that can be adapted to radio networks, the *Ad hoc* topology is the most attractive since it consists to connect mobile nodes without pre-existing infrastructure. When some nodes are not directly in range of each other there is a need of packet relaying by intermediate nodes. The working group MANet of Internet Engineering Task Force (IETF) is standardizing routing protocol for *ad hoc* wireless networking under Internet Protocol (IP). In MANet every node is a potential router for other nodes. The task of specifying a routing protocol for a mobile wireless network is not a trivial one. The main problem encounterd in mobile networking is the limited bandwidth and the high rate of topological changes and link failure caused by node movement. In this case the classical routing protocol as Routing Internet Protocol (RIP) and Open Shortest Path First (OSPF) first introduced in ARPANET [1] are not adapted since they need too much control traffic and can only accept few topology changes per minute.

MANet working group proposes two kinds of routing protocols:

1. The reactive protocols;

2. the pro-active protocols.

The reactive protocols such as AODV [3], DSR [2], and TORA [4], do not need control exchange data in absence of data traffic. Route discovery procedure is invoked on demand when a source has a new connection pending toward a new destination. The route discovery procedure in general consists into the flooding of a *query* packet and the return of the route by the destination. The exhaustive flooding can be very expensive, thus creating delays in route establishment. Furthermore the route discovery via flooding does not guarantee to create optimal routes in terms of hop-distance.

The pro-active protocols such as Optimized Link State Routing (OLSR) [5], TBRPF [6], need periodic update with control packet and therefore generates an extra traffic which adds to the actual data traffic. The control traffic is broadcasted all over the network via optimized flooding. Optimized flooding is possible since nodes permanently monitor the topology of the network. OLSR uses multipoint relay flooding which very significantly reduce the cost of such broadcasts. Furthermore, the node have permanent dynamic database which make optimal routes immediately available on demand. The protocol OLSR has been adapted from the *intra-forwarding* protocol in HIPERLAN type 1 standard [7]. Most of the salient features of OLSR such as multipoint relays and link state routing are already existing in the HIPERLAN standard.

The aim of the present paper is to analyze the performance of the multipoint relaying concept of OLSR under two models of network: the random graph model and the random unit graph model. The paper is divided into four main sections. The first section summarizes the main feature of OLSR protocol. The second section introduces and discusses the graph models. The third section develops the performance analysis of OLSR with respect to the graph models. A fourth section discuses more specifically about the comparison between MPR flooding and other known techniques of flooding optimization.

2 The Optimized Link State Routing protocol

2.1 Non optimized link state algorithm

Before introducing the optimized link state routing we make a brief reminder about non optmized link state such as OSPF. In an *ad hoc* network, we call link, a pair of two nodes which can hear each other. In order to achieve unicast transmission, it is important here to use bidirectionnal link (IEEE 802.11 radio LAN standard requires a two way packet transmission). However due to sensitivity of power discrepancies, unidirectional links can arise in the network. The use of unidirectional links is possible but require different protocols and is omitted here. Each link in the graph is a potential hop for routing packets. The aim of a link state protocol is that each node has sufficient knowledge about the existing link in the network in order to compute the shortest path to any remote node.

Each node operating in a link state protocol performs the two following tasks:

- **Neighbor discovery:** to detect the adjacent links;

- **Topology broadcast:** to advertize in the whole network about important adjacent links.

By important adjacent links we mean a subset of adjacent links that permit the computation of the shortest path to any destination.

The simplest neighbor discovery consists for each node to periodically broadcast full hello packets. Each full hello packet contains the list of the heared neighbor by the node. The transmission of hello packets is limited to one hop. By comparing the list of heared nodes each host determine the set of adjacent bidirectional links.

A non optimized link state algorithm performs topology broadcast simply by periodically flooding the whole network with a topology control packet containing the list of all its neighbor nodes (i.e. the heads of its adjacent links). In other words, all adjacent links of a node are important. By flooding we mean that every node in the network re-broadcast the topology control packet upon reception. Using sequence number prevents the topology control packet to be retransmitted several times by the same node. The number of transmissions of a topology control packet is exactly N, when N is the total number of nodes in the network, and when retransmission and packet reception are error free.

If h is the rate of hello transmission per node and τ the rate of topology control generation, then the actual control overhead in terms of packet transmitted of OSPF is

$$hN + \tau N^2 \tag{1}$$

As the hello packets and the topology control packet contains the IP addresses of originator node and neighbor nodes, the actual contro overhead can be expressed in terms of IP addresses unit. The overhead in IP address units is

$$hNM + \tau N^2 M \tag{2}$$

where M is the average number of adjacent links per node. If M is of the same order than N then the overhead is cubic in N. Notice that the topology broadcast overhead is one order of magnitude larger than the neighbor discovery overhead.

Notice that for non-optimized link state routing the hello and topology control packet can be the same.

2.2 OLSR and MultiPoint Relay nodes

The Optimized Link State Routing protocol is a link state protocol which optimizes the control overhead via two means:

1. the important adjacent links are limited to MPR nodes;

2. the flooding of topology control packet is limited to MPR nodes (MPR flooding).

The concept of MultiPoint Relay (MPR) nodes has been introduced in [7]. By MPR set we mean a subset of the neighbor nodes of a host which covers the two-hop neighborhood of the host. The smallest will be the MPR set the more efficient will be the optimization. We give a more precise definition of the multipoint relay set of a given node A in the graph. We define the neighborhood of A as the set of nodes which have an adjacent link to A. We define the two-hop neighborhood of A as the set of nodes which have an invalid link to A but have a valid link to the neighborhood of A. This information about two-hop neighborhood and two-hop links are made available in hello packets, since every neighbor of A periodically broadcasts their adjacent links. The multipoint relay set of A (MPR(A)) is a subset of the neighborhood of A which satisfies the following condition: every node in the two-hop neighborhood of A must have a valid link toward MPR(A).

The smaller is the Multipoint Relay set is, the more optimal is the routing protocol. [13] gives an analysis and examples about multipoint relay search algorithms. The MPR flooding can be used for any kind of long hole broadcast transmission and follows the following rule:

> *A node retransmits a broadcast packet only if it has received its first copy from a node for which it is a multipoint relay.*

[7] gives a proof that such flooding protocol (selective flooding) eventually reaches all destinations in the graph.[7] also gives a proof that for each destination in the network, the subgraph made of all MPR links in the network

and all adjacent links to host A contains a shortest path with respect to the original graph.

Therefore the multipoint relays improve routing performance in two aspects:

1. it significantly reduces the number of retransmissions in a flooding or broadcast procedure;

2. it reduces the size of the control packets since OLSR nodes only broadcast its multipoint relay list instead of its whole neighborhood list in a plain link state routing algorithm.

In other words if D_N is the average number of MPR links per node and R_N the average number of retransmission in an MPR flooding, then the control traffic of OLSR is, in packet transmitted:

$$hN + \tau R_N N \ , \tag{3}$$

and, in IP addresses transmitted:

$$hMN + \tau R_N D_N N \ . \tag{4}$$

Notice that when the nodes selects all their adjacent links as MPR links, we have $D_N = M$ and $R_N = N$: we have the overhead of a full link state algorithm. However we will show that straightforward optimizations make $D_N \ll M$ and $R_N \ll N$ gaining several orders of magnitude in topology broadcast overhead. Notice that the neighbor discovery overhead is unchanged. Summing both overhead we may expect that OLSR has an overhead reduced of an magnitude order with respect to full link state protocol.

The protocol as it is proposed in IETF may differ to some details from this very simple presentation. The reason is for second order optimization with regards to mobility for example. For example hosts in actual OLSR do not advertize their MPR set but their MPR selector set, i.e. the subset of neighbor nodes which have selected this host as MPR.

2.3 MPR selection

Finding the optimal MPR set is an NP problem as proven in [8]. However there are very efficient heuristics. Amir Qayyum [13] has proposed the following one:

1. select as MPR, the neighbor node which has the largest number of links in the two-hop neighbor set;
2. remove this MPR node from the neighbor set and the neighbor nodes of this MPR node from the two-hop neighbor set;
3. the previous steps until the two-hop neighbor set is empty.

An ultimate refinement is a prior operation which consists into detecting in the two-hop neighbor the node which have a single parent in the neighbor set. These parents are selected as MPR and are eliminated from the neighbor set, and their neighbor are eliminated in the two-hop neighbor set.

It is proven in [8, 10] that this heuristic is optimal by a factor $\log M$ where M is the size of the neighbor set (i.e. the heuristical MPR set is at most $\log M$ times larger than the optimal MPR set).

2.4 Deterministic properties of OLSR protocol

The aim of this section is to show some properties of OLSR protocol which are independent of the graph model. Basically we will prove the correct functioning of OLSR protocol. In particular we will prove that the MPR flooding actually reaches all destination and that the route computed by OLSR protocol (and actually used by data packets) have optimal length.

To simplify our proof we call chain of nodes, any sequence of nodes $A_1, \ldots A_n$ such that each pair (A_i, A_{i+1}) are connected by a (bidirectionnal) link.

Theorem 1 *When at each hop broadcast packets are received error-free by all neighbor nodes, the flooding via MPR reaches all destinations.*

Remark When the transmissions are prone to errors, then there is no guarantee of correct delivery of the broadcast packet to all destinations, even with a full flooding retransmission process. Amir Qayyum, Laurent Viennot Anis Laouiti *et al.* show in [10] the effect of errors on full flooding and MPR flooding. It basically show that MPR flooding and full flooding have similar reliability.
Proof: Since we assume error free broadcast transmission, any one hop broadcast reaches all neighbor nodes.

Let assume a broadcast initiated by a node A. Let B be an arbitrary node in the network. Let k be smallest number of hops between B and the set of nodes which eventually receive the broadcast message. We will show that actually $k = 0$; i.e, B receives the broadcast message.

Let assume *a contrario* that $k > 0$. Let F be the first node at distance $k+1$ from node B which retransmitted the broadcast message. We know that this node exists since there is a node at distance k from node B which received the broadcast message. Let $F_1, \ldots, F_k$ the chain of nodes that connects node B to node F in $k+1$ hops. Node F_{k-1} is at distance $k-1$ to node B (when $k = 1$, node F_{k-1} is node B). Node F_{k-1} is also in the two-hop neighborhood of node F. Let F' be an MPR of node F which is neighbor of F_{k-1}. Since node F' receives its first copy of the broadcast message from its MPR selector F, then F' must retransmit the broadcast message, which contradicts the definition of k. ■

We now prove that the route computed by OLSR protocol have optimal length. It is easy to see that this property is the corollary of the following theorem.

Theorem 2 *If two nodes A and B are at distance $k+1$, then there exists a chain of nodes $F_1, \ldots, F_k$ such that the three following points hold:*

- *(i) node F_k is MPR of node B;*
- *(ii) node F_i is MPR of node F_{i+1};*
- *(iii) node F_1 is a neighbor node of node A.*

Proof: The proof goes by induction. The property is trivial when $k = 0$. When $k = 1$, node A is in the two-hop neighborhood of B. Thus there exists an MPR node F of node B which is at distance one hop of node A, which proves the property for $k = 1$. Let now assume that the property is true until a given value k. We will prove that the property is also true for value $k+1$. Let assume a node B at distance $k+2$ of node A. There exists an MPR node F of node B which is at distance $k+1$ of A (to be convinced, there exists a node F' which is at distance k from A and at distance 2 from B and node F can be one of the MPR nodes of B that cover F'). By the recursion hypothesis there exists a chain $F_1, \ldots, F_k$ which connects A to F such that:

- (i) node F_k is MPR of node F;
- (ii) node F_i is MPR of node F_{i+1};
- (iii) node F_1 is a neighbor node of node A.

The same property holds for the chain $F_1, \dots, F_k, F$ which connects A to B. The property holds for $k+1$.■

3 The graph models

The modelization of ad hoc mobile network is not an easy task. Indeed the versatility of radio propagation in presence of obstacles, distance attenuation and mobility is the source of incommensurable difficulties. In passing, one should notice that mobility not only encompasses host mobility but also the mobility of the propagation medium. For example when a door is open in a building, then the distribution of links change. If a truck passes between two hosts it may switch down the link between them. In this perspective building a realistic model that is tractable by analysis is hopeless. Therefore we will focus on models dedicated to specific scenarios.

There are two kinds of scenarios: the indoor scenarios and the outdoor scenarios. For the indoor scenario we will use the random graph model. For the outdoor scenarios we will use the random unit graph model. The most realistic model lies somewhere between the random graph model and the random unit graph model.

3.1 The random graph model for indoor networks

In the following we consider a wireless indoor network made of N nodes. The links are distributed according to a random graph with N vertices and link probability is p. In other words, a link exists between two given nodes with probability p. Link's existence are independent from one pair of nodes to another. Figure 1 shows an example of a random graph with $(N, p) = (10, 0.7)$, the nodes have been drawn in concentric mode just for convenience.

The random graph model implicitly acknowledges the fact that in an indoor network, the main cause of link obstruction is the existence of random

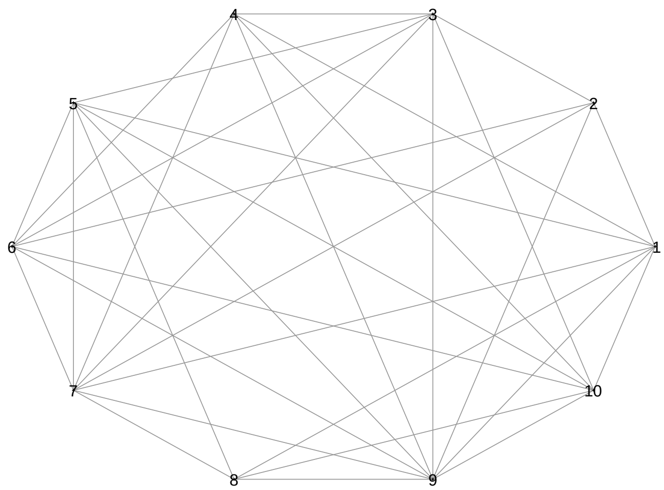

Figure 1: A random graph with $n = 10$ and $p = 0.7$, generated by Maple

obstacle (wall, furniture) between any pair of nodes. The fact that the links are independently distributed between node pairs assumes that these obstacles are independly distributed with respect to node position, which of course is never completely true. However the random graph model is the simplest satisfactory model of indoor radio network and provides excellent results as a starting point.

When the network is static, then the graph does not change during the time. It is clear that nodes does not frequently change position in indoor model, but the propagation medium can vary. In this case the random graph may vary with the time. One easy way to model time variation is to assume random and independent link lifetime. For example, one can define μ as link variation rate, i.e. the rate at which each link may come down or up. During an interval $[t, t + dt]$ a link can change its status with probability μdt, i.e. it takes status "up" or "down" with probability p, independly of its previous status. The effect of mobility won't be investigated in the present paper.

3.2 The random unit graph model for outdoors networks

To explain this kind of graph it suffices to refer to a very simple example. Let L be a non-negative number and let us define a two-dimensional square of size $L \times L$ unit lengths. Let consider N nodes uniformly distributed on this square. The unit graph is the graph obtained by systematically linking pairs nodes when their distance is smaller or equal to the unit length. This model of graph is well adapted to outdoor networks where the main cause of link failure is the attenuation of signal by the distance. In this case the area where a link can be established with a given host is exactly the disk of radius the radio range centered on the host. However the presence of obstacle may give a more twisted shape to the reception area (that may not be single connected).

Figure 2 and 3 respectively show the two steps of the build up of a random unit graph of dimension two. The first step is the uniform distribution of the points on the rectangle area. The second step is the link distribution between node pairs according to their distance.

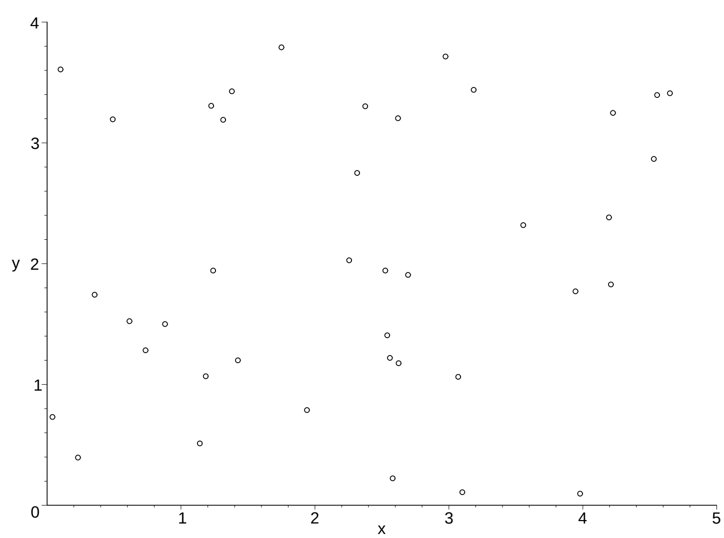

Figure 2: forty points uniformly distributed on a 5×4 rectangle

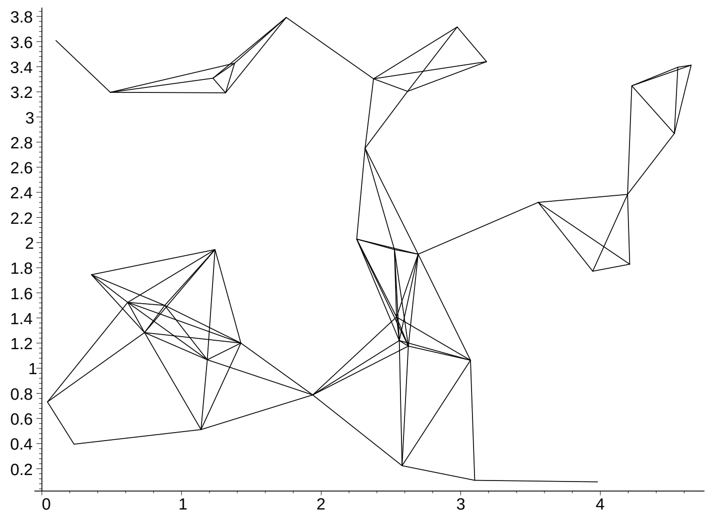

Figure 3: The random unit graph derived from the forty points locations of figure 2

The reception area may also change with the time, due to node mobility, obstacle mobility, noise or actual data traffic. In the present paper we will assume that the network is static.

Of course, the unit graph can be defined on other space than the plane. For example a unit graph can be defined on a 1D segment, modeling a mobile network made of cars on a road. It can be a cube in the air, modeling a mobile network made of airplanes, for example.

4 Analysis of OLSR in the random graph model

4.1 Route lengths

Most pro-active protocols (like OLSR) have the advantage to deliver optimal routes (in term of hop number) to data transfers. The analysis of optimal routes is very easy in random graph models since a random graph tends to be of diameter 2 when N tends to infinity with fixed p.

Theorem 3 *The optimal route between two random nodes in a random graph, when N tends to infinity,*

(i) is of length 1 with probability p;

(ii) or of length 2 with probability $q = 1 - p$.

Proof: Point (i) is an easy consequence of the random graph model. For point (ii) we consider two nodes, node A and node B, which are not at distance 1 (which occurs with probability $1-p$). We assume *a contrario* that these nodes are not at distance 2, and we will prove that would occur with a probability p_3 which exponentially tends to zero when N increases. If the distance between A and B is greater than 2, then for each of the $N-2$ remaining nodes in the network either

1. the link to A is down;

2. or the link to B is down.

For every remaining node the above event occurs with probability $1 - p^2$, therefore $p_3 = (1 - p^2)^{N-2}$, which proves the theorem.

4.2 Multipoint relay flooding

Theorem 4 *For all $\varepsilon > 0$, the optimal MPR set size D_N of any arbitrary node is smaller than $(1+\varepsilon)\frac{\log N}{-\log q}$ with probability tending to 1 when N tends to infinity.*

Proof: We assume that a given node A randomly selects k nodes in its neighborhood and we will fix the appropriate value of k which makes this random set a multipoint relay set. The probability that any given other point in the graph be not connected to this random set is $(1-p)^k$. Therefore the probability that there exists a point in the graph which is not connected via a valid link to the random set is smaller than $N(1-p)^k$. Taking $k = (1+\varepsilon)\frac{\log N}{-\log q}$ for some $\varepsilon > 0$ makes the probability tending to 0. ∎

Notice that $D_N = O(\log N)$ which very favorably compares to the size of the whole host neighborhood (which is in average pN) and considerably reduces the topology broadcast.

Theorem 5 *The broadcast or flooding via multipoint relays takes in average a number R_N of retransmissions smaller than $(1+\varepsilon)\frac{\log N}{-p\log q}$.*

Proof: First we notice that this average number favorably compares to the unrestricted flooding needed in plain links state routing algorithms which exactly needs N retransmissions per flooding.

Let us consider a flooding initiated by an arbitrary node. We sort the retransmission of the original message according to their chronological order. The 0-th retransmission corresponds to the source of the broadcast. We call m_k the size of the multipoint relay set of the k-th retransmitter. We assume that each of the m_k multipoint node of the kth transmitter are chosen randomly as in the proof of theorem 4. The probability that a given multipoint relay points of the k-th transmitter did not receive a copy of the broadcast packet from the k first retransmissions is $(1-p)^k$. Therefore the average number of new hitted multipoint relays which will have to actually retransmit the broadcast message after its kth retransmission is $(1-p)^k m_k$. Consequently the average total number of retransmissions does not exceed $\sum_{k\geq 0}(1-p)^k m_k$.

Using the upper bound $m_k \leq (1+\varepsilon)\frac{\log N}{-\log q}$, the average number of retransmission is upper bounded by $(1+\varepsilon)\frac{\log N}{-\log q}\sum_{k\geq 0}(1-p)^k = (1+\varepsilon)\frac{\log N}{-p\log q}$, which ends the proof of the theorem.

Corollary 1 *The cost of OLSR control traffic for topology broadcast in the random graph model is $O(N(\log N)^2)$ compared to $O(N^3)$ with plain link state algorithm.*

Remark: Notice that the neighbor sensing in $O(N^2)$ is now the dominant source of control traffic overhead.

5 Analysis of OLSR in the random unit graph

5.1 Analysis in 1D

A 1D unit graph can be made of N nodes uniformly distributed on a strip of land whose width is smaller than the radio range (set as unit length). We assume that the length of the land strip is L unit length.

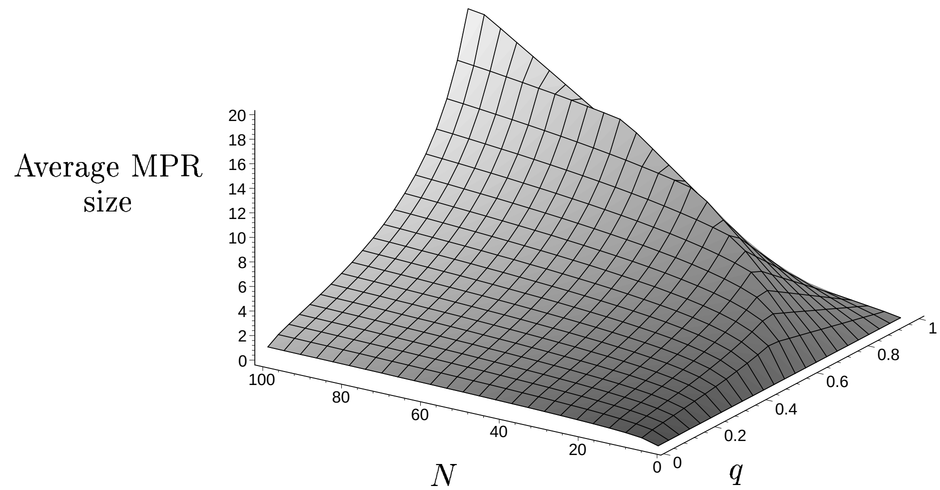

Figure 4: average multi-point relay set size in (p, N), $q = 1 - p$

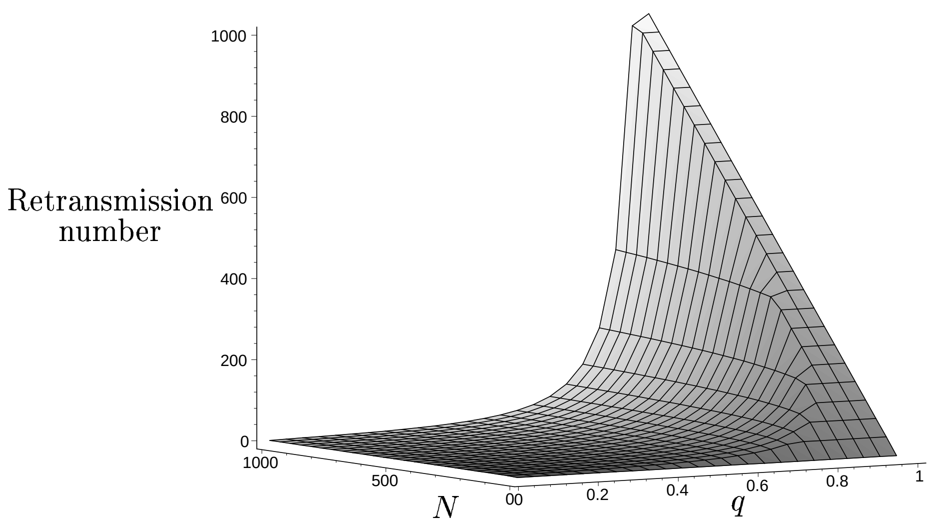

Figure 5: average number of retransmissions in a multi-point relay flooding in (q, N)

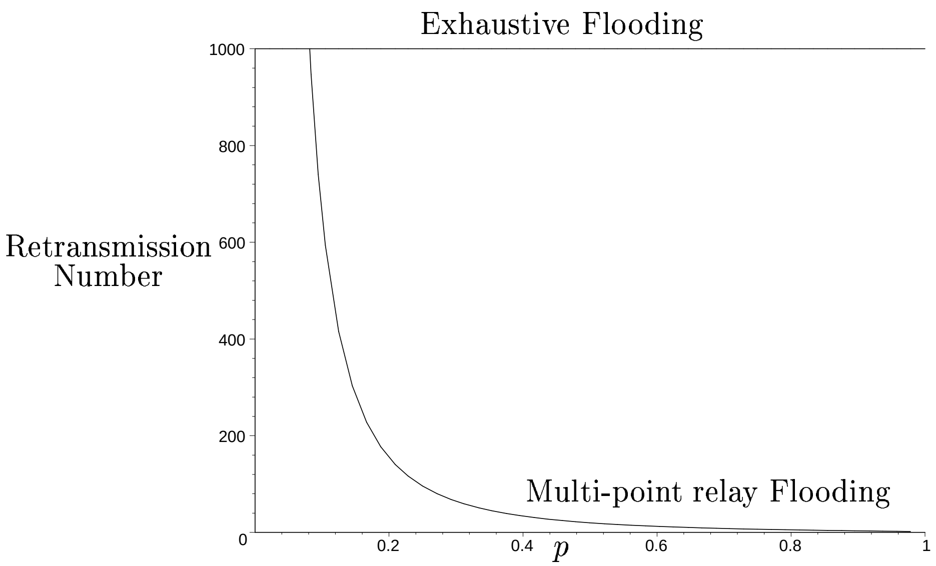

Figure 6: average number of retransmissions in multi-point relay flooding with $N = 1000$ and p variable

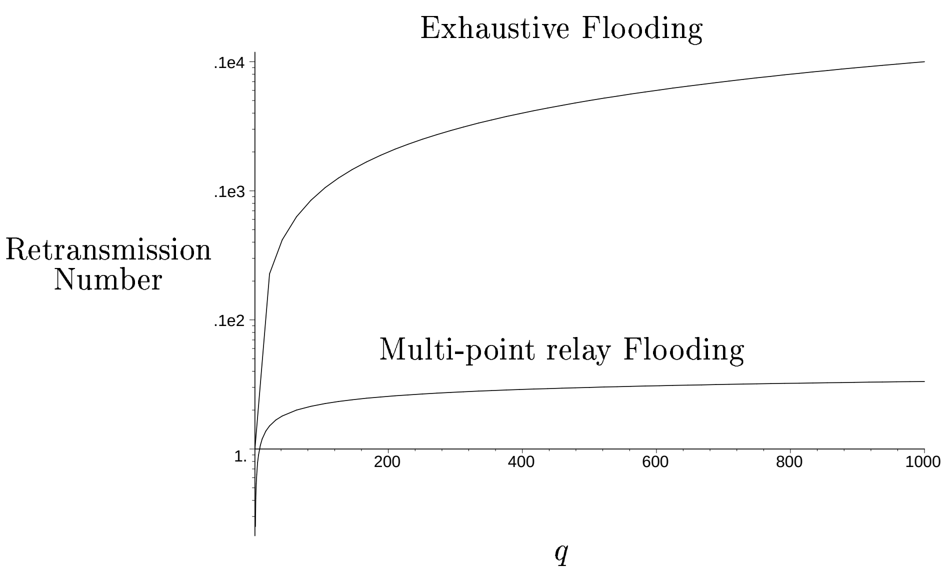

Figure 7: average number of retransmissions in multi-point relay flooding with N variable and $p = 0.9$, logarithmic scale.

Theorem 6 *The size of the MPR set D_N of a given host is 1 when the host is at one radio hop to one end of the strip, and 2 otherwise.*

Proof: The proof is rather trivial. The heuristic finds the nodes which cover the 2-hop neighbors of the host. These nodes are the two nodes which are the farther in the neighborhood of the host (one on the left side, the other one on the right side). These two nodes cover the whole 2-hop neighborhood of the host and make the optimal MPR set. Notice that only one MPR suffices when the strip ends in the radio range of the host.■

Theorem 7 *The MPR flooding of a broadcast message originated by a random node takes $R_N = \lfloor L \rfloor$ retransmission of the message when N tends to infinity and L is fixed.*

Proof: The distance between the host and its MPR tends to be equal to one unit length when the density increases.■

Notice this is assuming an error free retransmission. In case of error, the retransmission stops at the first MPR which does not receive correctly the message. In order to cope with this problem one may have to add redundancy in the MPR set which might be too small with regard to this problem.

Notice that these figures favorably compare with plain link state where $D_N = M = N/L$ and $R_N = N$.

5.2 Analysis in 2D

The analysis in 2D is more interesting because it gives less trivial results. We need the following elementary lemma about geometry. The proof is left to the reader.

Lemma 1 *Let consider two disks K_1 and K_2 of respective radius 1 and 2 centered on origin O. Let two points A and B on the border of K_1 separated by an angle θ (measured from origin). Let $\mathcal{A}(\theta)$ be the area of the set of points of K_2 such that*

- *the points are not in K_1;*
- *the points are in the sector of origin O, limited by A and B;*

- *the points are at distance greater than 1 from both A and B.*

When $\theta \leq 2\pi/3$ then $\mathcal{A}(\theta) = \theta - \sin\theta$. Otherwise $\mathcal{A}(\theta) = \mathcal{A}(2\pi/3) + 3 \times (\theta - 2\pi/3)$.

Theorem 8 *When L is fixed and N increases, then the average size of the MPR set, D_N tends to be smaller than $3\pi(N/(3L^2))^{1/3} = 3\pi(M/(3\pi))^{1/3}$.*

Notice that this figure compares favorably with plain link state where $D_N = M = N/L^2$.

Proof: We only give a sketch of the proof. We assume that L is not too small (greater than 4). Let consider an arbitrary host located on the square. Let K_1 be the disk of radius 1 centered on the host, and K_2 be the disk of radius 2 also centered on the host. We know that the one-hop neighborhood of the host are located in disk K_1, and the two-hop neighborhood is located in the set $K_2 - K_1$. To simplify, we assume that disk K_2 does not intercept the square border. The MPR selection heuristic naturally leads to select MPRs in the limit of the radio range of the host, i.e. the closer to the border of disk K_1. Indeed, this is where the neighbors cover the most the two-hop neighborhood of the host (i.e $K_2 - K_1$). When the network density is high we can assume that the MPRs are actually on the border of K_1.

Let consider k MPRs candidates identified by $B_1, \ldots, B_k$. We suppose that the B_i considered in increasing order are located clockwisely on the unit circle. Let θ_i be the angle made by the sector limited by B_i and B_{i+1}, with the boundary case θ_k is the angle made by the sector limited by B_k and B_1. We have $\theta_1 + \cdots + \theta_k = 2\pi$.

In order to make $\{B_1, \ldots, B_k\}$ a suitable MPR set, one needs that the union of the disks $K(B_i)$ of radius 1 and center B_i contains the whole two-hop neighborhood of the host. This condition is fullfilled when the uncovered set $K_2 - K_1 - \cup_{i=1}^{i=k} K(B_k)$ does not contain any node of the network. If it is not the case therefore one has to add to $\{B_1, \ldots, B_k\}$ extra neighbor nodes that covers the nodes in $K_2 - K_1 - \cup_{i=1}^{i=k} K(B_k)$.

The area of the uncovered set is exactly $\mathcal{A}_k = \mathcal{A}(\theta_1) + \cdots + \mathcal{A}(\theta_k)$, therefore the average number of nodes in this area is $\mathcal{A}_k \times D$, where D is the density of the network ($D = N/L^2$). Therefore the average number of extra nodes is smaller than $\mathcal{A}_k D$. Therefore $k + A_k D$ is an upperbound of D_N. Notice that for k given, quantity $\mathcal{A}_k$ is minimal when the θ_i's are all equal, namely when $\theta_i = 2\pi/k$ for all i. In this case $\mathcal{A}_k = k\mathcal{A}(2\pi/k)$ and

$$D_N \leq k + k\mathcal{A}(2\pi/k)D \; . \tag{5}$$

Using $\mathcal{A}(\theta) \approx \theta^3/6$, we have $D_N \leq k + (2\pi)^3 D/(6k^2)$. The right-hand side attains its minimum for $k = 2\pi(D/3)^{1/3}$ and it comes $D_N \leq 3\pi(D/3)^{1/3}$.■

Figure 8 displays simulation results for dimension 2. The heuristic has been applied to the central node of a random 4×4 unit graph. The convergence in $M^{1/3}$ is clearly shown. Notice that in this very case the upper bound of D_N is at least greater by a factor 2 than actual values obtained by simulations. Figure 9 summarizes the results obtained for quantity D_N in the random graph model for dimension 1 and 2. The results for dimension 2 have been simulated.

Theorem 9 *The MPR flooding of a broadcast message originated by a random node takes $R_N = O((NL^4)^{1/3})$ retransmissions of the message when N tends to infinity and L is fixed.*

Proof: There is no complete proof of this theorem. We can give a sketch of hint. We call MPR hit when a retransmission is received by a MPR point for the first time. The hitted MPRs will have to retransmit the broadcast message. At each retransmission of the broadcast packet there is an area size of order $O(1)$ which is added to the already covered size. In the same time there are $O(M^{1/3})$ new additional MPR hits. This new area contains $O(M)$ points. Therefore to cover the whole area there would be a need of $O(N/M)$ retransmissions with consequently $O(N/M \times (M)^{1/3})$ MPR hitted. When the area is completely covered there is no possibility of new MPR hitted and flooding stops. The total number of retransmissions equals the number of MPR hits which is $O(NM^{-2/3})$. The estimate $M = O(N/L^2)$ terminates the proof.■

5.3 Comparison with dominating set flooding

In [11] Wu and Li introduced the concept of dominating set. They introduced two kinds of dominating set that we will call, the rule 1 dominating set and the rule 2 dominating set. In this section we establish quantative comparisons

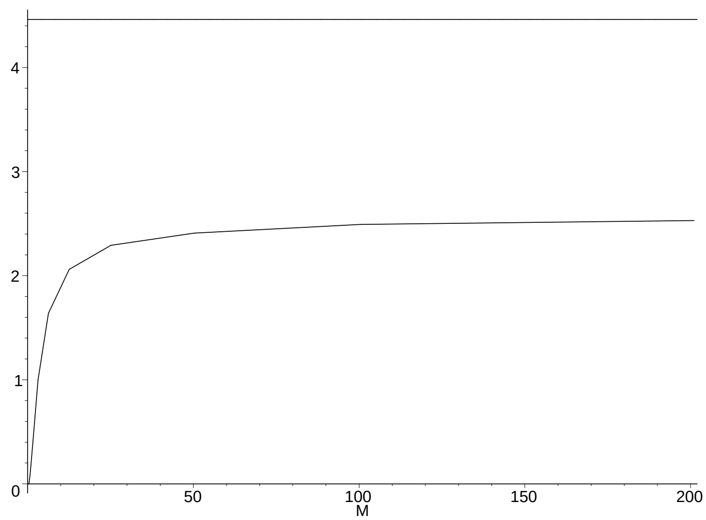

Figure 8: Bottom: simulated quantity $D_N/M^{1/3}$ versus the number of neighbor M for the central position in a 4×4 random unit graph,top: upper bound obtained in theorems.

between the performance of dominating set flooding and MPR flooding. In particular we will show that dominating set floodings does not outperform significantly full flooding in random graph models and in random unit graph of dimension 2 and higher. Rule 1 dominating set does not outperform significantly full flooding in random unit graph model of dimension 1. MPR flooding outperforms both dominating set flooding in any graph models studied in this paper.

The dominating set flooding consists into restricting the retransmission of a broadcast message to a subset of nodes, called the dominating set. Rule 1 and rule 2 consist into two different rules of dominating set selection. The rules consist into compairing neighbor sets (for example by checking hellos). For a node A we denote by $\mathcal{N}(A)$, the neighbor set of node A.

In rule 1, a node A does not belong to the dominating if and only if there exists a neighbor B of A such that

1. B is in the dominating set;

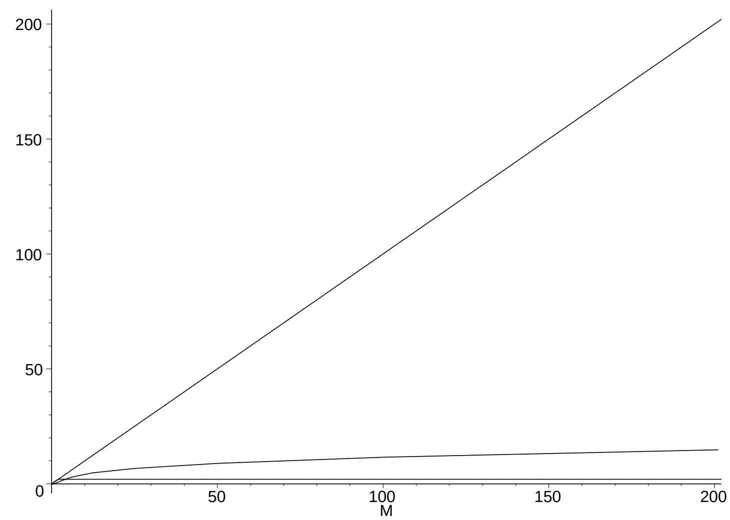

Figure 9: unit graph model from bottom to top, average number of MPR for 1D, 2D and full links state protocol versus the average number of neighbor nodes M.

2. the IP address of B is higher than the IP address of A;
3. $\mathcal{N}(A) \subset \mathcal{N}(B)$.

In this case one says that B dominates A in rule 1.

In rule 2, a node A does not belong to the dominating if and only if there exist two neighbor B and C of A such that

1. B and C are in the dominating set;
2. nodes B and C are neighbors;
3. the IP addresses of B and C are both higher than the IP address of A;
4. $\mathcal{N}(A) \subset \mathcal{N}(B) \cup \mathcal{N}(C)$.

In this case one says that (B, C) dominates A in rule 2.

We first, look at the performance of dominating set flooding in the random graph model (N, p).

Theorem 10 *The probability that a node in a random graph (N, p) does not belong to the dominating set is*

- *smaller than $N(1-(1-p)p)^N$ in rule 1;*
- *smaller than $N^2(1-(1-p)^2p)^N$ in rule 2.*

Proof: We first concentrate on rule 1. Let us consider that a given node A has k neighbors. This occurs with probability $p^k(1-p)^{N-k}\binom{n}{k}$. The probability that another given node B is neighbor of A is p. The probability that the $k-1$ other neighbors of A are also neighbors of B is p^{k-1}. Therefore the probability that B is neighbor of A and $\mathcal{N}(A) \subset \mathcal{N}(B)$ is $\sum_{k=0}^{N} p^{2k}(1-p)^{N-k}\binom{n}{k}$ which is equal to $(1-(1-p)p)^N$. The probability that there exists at least one of the $N-1$ nodes other than A that dominates A in rule 1 is smaller than $N(1-(1-p)p)^N$.

The proof on rule 2 is similar. Given k neighbors to A, the probability that two given other nodes B and C are neighbors of A is p^2. The probability that $\mathcal{N}(A) \subset \mathcal{N}(B) \cup \mathcal{N}(C)$ is $(1-(1-p)^2)^{k-2}$. Therefore the probability that there exist a pair that dominates A in rule 2 is smaller than $N^2\sum_{k=0}^{N} p^{2k}(2-p)^{k-2}(1-p)^{N-k}\binom{n}{k}$ which is equal to $N^2(1-(1-p)^2p)^N/(2-p)$. ∎

Theorem 11 *In the random unit graph model of dimension 1, assuming independence between node location and node IP addresses, the probability that a node does not belong to the dominating set in rule 1 is smaller than $\frac{4}{M}$ and the average size of the dominating set in rule 2 is* $\max\{0, 2L-1\}$.

Remark: The density of the dominating set in rule 2 is twice than the density of retransmitters in MPR flooding when the network model is the random unit graph of dimension one.

Proof: Let consider a node A and a node B at distance y. The set $\mathcal{N}(A) - \mathcal{N}(B)$ is supported by a segment of length y. Therefore the probability that $\mathcal{N}(A) - N(B) = \emptyset$ is equal to e^{-yD} where D is the density of the network $(D = M/2)$. Therefore the unconditional probability that A does not belong to the dominating set in rule 1 is smaller than $2\int_0^1 e^{-yD}dy$.

The study of rule 2 is more intricate. It is clear that the node with the highest IP address is in the dominating set. The second highest IP address is also in the dominating set. The third highest IP address is also in the dominating set provided that it does not stand between the highest and the second highest and the latter nodes are at distance smaller than one of each other.

More generaly if $R(x)$ is the average size of the dominating set in a random unit graph in a segment of length x, then we have $R(x) = 0$ when $x < 1$, and when $x \geq 1$ one has

$$R(x) = 1 + \frac{1}{x}\int_0^x (R(y) + R(x-y))dy \tag{6}$$

which leads to the differential equation $xR'(x) = 1 + R(x)$ whose solution is $2x - 1$. ■

In random graph of dimension 2 and higher the probabilities that a node does not belong to the dominating set in rule 1 or in rule 2 are $O(1/D)$ since it is impossible to cover one unit disk with two unit disk that have different centers.

6 Conclusion and further works

We have presented a performance evaluation of OLSR mobile ad-hoc routing protocols in the random graph model and in the random unit graph model. The originality of the performance evaluation is that it is completely based on analytical methods (generating function, asymptotic expansion) and does not rely on simulation software. The random graph model is enough realistic for indoor or short range outdoor networks where link fading mainly comes from random obstacles. The random unit graph model is realistic for long range outdoor networks where link fading mainly comes from distance attenuation. In this case the random graph model can be improved by letting the parameter p depending on distance x between the nodes. The analytical derivation of the performances of the routing protocl in the distance dependent random graph will be subject of further works.

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